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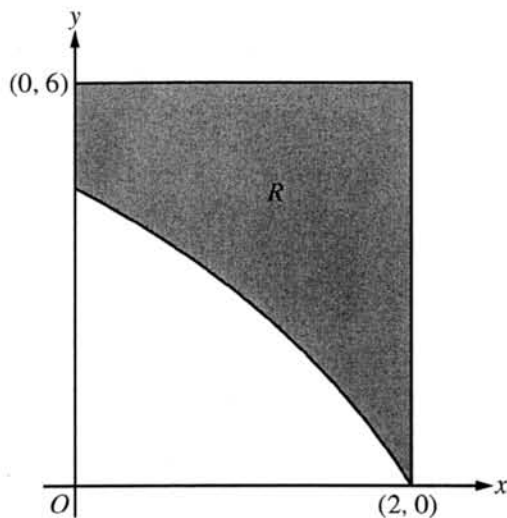
1A

CALCULUS BC
SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

$$\begin{aligned}
 SR &= 2 \times 6 - \int_0^2 4 \ln(3-x) dx \\
 &= \cancel{2} 12 - (5.183) \approx 6.817
 \end{aligned}$$

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Continue problem 1 on page 5.

Work for problem 1(b)

assum $y_1 = 4 \ln(3-x)$

$y_2 = 6$

$$V = \pi \int_0^2 \left[\frac{(8-y_1)^2}{2} - \frac{(8-y_2)^2}{2} \right] dx$$

$$= \pi \int_0^2 \left[(8-y_1)^2 - (8-y_2)^2 \right] dx$$

$$= \pi \int_0^2 \left[(8-4 \ln(3-x))^2 - (8-6)^2 \right] dx$$

$\approx 168.180.$

Work for problem 1(c)

$$V = \int_0^2 [6 - 4 \ln(3-x)]^2 dx.$$

$\approx 26.267.$

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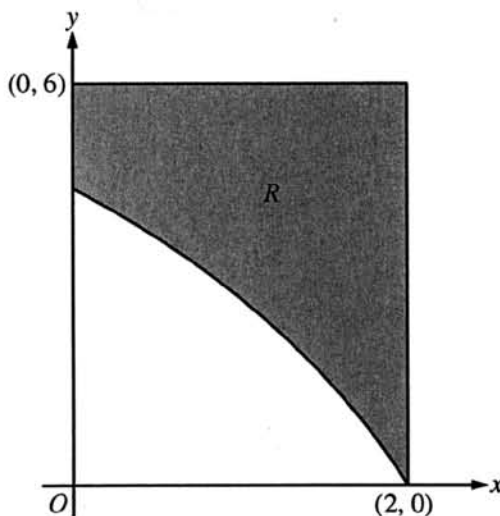
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CALCULUS AB
SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

$$\begin{aligned}
 R &= \int_0^2 6 - 4 \ln(3-x) \, dx \\
 &= \int_0^2 6 \, dx - \int_0^2 4 \ln(3-x) \, dx \\
 &= 6x \Big|_0^2 - 4 \int_0^2 \ln(3-x) \, dx \\
 &\quad \begin{array}{l} f = \ln(3-x) \quad f' = \frac{-1}{3-x} \\ g' = 1 \quad g = x \end{array} \\
 &= 12 - \left(4x \ln(3-x) \Big|_0^2 - \int_0^2 \frac{-x}{3-x} \, dx \right) \\
 &= 6.817 \text{ units}^2
 \end{aligned}$$

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Continue problem 1 on page 5.

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Work for problem 1(b)

$$\begin{aligned}
 V &= \pi \int_0^2 [8 - 4 \ln(3-x)]^2 - (8-6)^2 dx \\
 &= \pi \int_0^2 ([8 - 4 \ln(3-x)]^2 - 2^2) dx \\
 &= 168.180 \text{ units}^3
 \end{aligned}$$

Work for problem 1(c)

$$\begin{aligned}
 V &= \int_0^2 (2 \times 6)^2 - (2 \times 4 \ln(3-x))^2 dy \\
 &= \int_0^2 12^2 - (8 \ln(3-x))^2 dx \\
 &= 222.133 \text{ units}^3
 \end{aligned}$$

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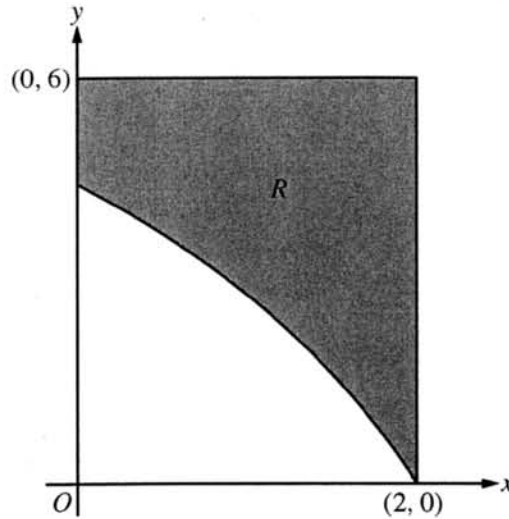
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CALCULUS BC
SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

$$A = \int_0^2 (6 - 4 \ln(3-x)) dx$$

with the help of the calculator, we type this in
and we get $A = \int_0^2 (6 - 4 \ln(3-x)) dx = 6.8176685$

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Continue problem 1 on page 5.

Work for problem 1(b)

Since ~~the~~^{it} is revolved about $l: y=8$
we use the washer method

$$V = \pi \int_0^2 (8 - 6^2)(8 - (4 \ln(3-x))^2) dx$$

With the calculator, we get

$$V = \pi \int_0^2 (8 - 6^2)(8 - (4 \ln(3-x))^2) dx = 41.059$$

Work for problem 1(c)

The side of the square shall be $y = 4 \ln(3-x)$
which makes the Area of the square as $A = (4 \ln(3-x))^2$

$$V = \pi \int_0^2 (4 \ln(3-x))^2 dx$$

We use the calculator to get

$$V = \pi \int_0^2 (4 \ln(3-x))^2 dx = 51.732$$

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AP[®] CALCULUS AB
2010 SCORING COMMENTARY (Form B)

Question 1

Sample: 1A

Score: 9

The student earned all 9 points.

Sample: 1B

Score: 6

The student earned 6 points: the global limits point, 2 points in part (a), 3 points in part (b), and no points in part (c). In part (a) the student earned both points and the global limits point. The student's intermediate work includes a misplaced 4, but the correct numerical answer was treated as a restart since this was the calculator portion of the exam. In part (b) the student's work is correct. In part (c) the student does not use square cross sections and was not eligible for any points.

Sample: 1C

Score: 3

The student earned 3 points: the global limits point, 2 points in part (a), no points in part (b), and no points in part (c). In part (a) the student earned the global limits point and has correct work. In part (b) the student attempts to find the volume using washers, but the work is incorrect. In part (c) the student uses an incorrect width for the area of the square cross section and includes a factor of π .

AP[®] CALCULUS AB
2010 SCORING GUIDELINES (Form B)

Question 2

The function g is defined for $x > 0$ with $g(1) = 2$, $g'(x) = \sin\left(x + \frac{1}{x}\right)$, and $g''(x) = \left(1 - \frac{1}{x^2}\right)\cos\left(x + \frac{1}{x}\right)$.

- (a) Find all values of x in the interval $0.12 \leq x \leq 1$ at which the graph of g has a horizontal tangent line.
 (b) On what subintervals of $(0.12, 1)$, if any, is the graph of g concave down? Justify your answer.
 (c) Write an equation for the line tangent to the graph of g at $x = 0.3$.
 (d) Does the line tangent to the graph of g at $x = 0.3$ lie above or below the graph of g for $0.3 < x < 1$? Why?

- (a) The graph of g has a horizontal tangent line when $g'(x) = 0$.
 This occurs at $x = 0.163$ and $x = 0.359$.

2 : $\begin{cases} 1 : \text{sets } g'(x) = 0 \\ 1 : \text{answer} \end{cases}$

- (b) $g''(x) = 0$ at $x = 0.129458$ and $x = 0.222734$
 The graph of g is concave down on $(0.1295, 0.2227)$
 because $g''(x) < 0$ on this interval.

2 : $\begin{cases} 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

- (c) $g'(0.3) = -0.472161$
 $g(0.3) = 2 + \int_1^{0.3} g'(x) dx = 1.546007$
 An equation for the line tangent to the graph of g is
 $y = 1.546 - 0.472(x - 0.3)$.

4 : $\begin{cases} 1 : g'(0.3) \\ 1 : \text{integral expression} \\ 1 : g(0.3) \\ 1 : \text{equation} \end{cases}$

- (d) $g''(x) > 0$ for $0.3 < x < 1$
 Therefore the line tangent to the graph of g at $x = 0.3$ lies
 below the graph of g for $0.3 < x < 1$.

1 : answer with reason

Work for problem 2(a)

Horizontal tangent line : $g'(x) = 0$

$$\circ \circ \sin\left(x + \frac{1}{x}\right) = 0$$

$$x = 0.163 \text{ \& } 0.359$$

There exists horizontal tangent lines
at $x = 0.163$ and $x = 0.359$ *

Work for problem 2(b)

$g''(x) < 0$, g concaved down.

$$\circ \circ g''(x) = \left(1 - \frac{1}{x^2}\right) \cos\left(x + \frac{1}{x}\right) < 0$$

$$g''(x) < 0 \text{ on } (0.129, 0.223)$$

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Continue problem 2 on page 7.

Work for problem 2(c)

$$g'(0.3) = -0.472$$

$$\int_{0.3}^1 \sin\left(\pi + \frac{1}{x}\right) dx = 0.45399$$

$$g(0.3) = 2 + (-0.45399) \\ = 1.546$$

$$y - 1.546 = -0.472(x - 0.3)$$

$$y = -0.472x + 1.688 \quad \neq$$

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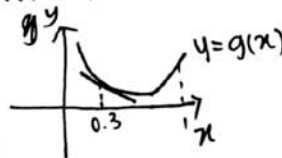
Work for problem 2(d)

At $x = 0.3$

$$g''(0.3) = 8.913 > 0 \quad g'(0.3) = -0.472 < 0$$

$\therefore g$ is concave up on $x = 0.3$ * g is concave up on $(0.3, 1)$ as $g''(x)$ remains positive on $(0.3, 1)$

As such the tangent of graph g at $x = 0.3$ will lie below the graph of g



GO ON TO THE NEXT PAGE.

Work for problem 2(a)

$$g'(x) = 0$$

$$0 = \sin\left(x + \frac{1}{x}\right)$$

$$x = 0.163$$

$$x = 0.359$$

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Work for problem 2(b)

g is concave down on the interval $(0.129, 0.223)$ because g' is decreasing on this interval and $g'' < 0$ on this interval.

$$g'' = 0$$

$$x = 0.129$$

$$x = 0.223$$



Continue problem 2 on page 7.

Work for problem 2(c)

$$m = g'(0.3) = \sin\left(0.3 + \frac{1}{0.3}\right) = -0.472161$$

$$(1, 2) \quad y = mx + b$$

$$2 = (-0.472161)(1) + b$$

$$2.47216 = b$$

$$y = -0.472x + 2.472$$

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Work for problem 2(d)

The line tangent to g at $x = 0.3$ lies below the graph of g for $0.3 < x < 1$ because on the interval $0.3 < x < 1$ g is concave up.

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Work for problem 2(a)

\therefore It has horizontal tangent line

$$\therefore g'(x) = 0$$

$$\therefore \sin\left(x + \frac{1}{x}\right) = 0$$

$$\therefore x = 0.163 \text{ or } 0.359$$

\therefore When x is equal to 0.163 or 0.359, the graph of g has a horizontal tangent

Work for problem 2(b)

$\therefore g$ is concave down

$$\therefore g'' < 0$$

$$\therefore \left(1 - \frac{1}{x^2}\right) \cos\left(x + \frac{1}{x}\right) < 0$$

$\therefore \left(1 - \frac{1}{x^2}\right) \cos\left(x + \frac{1}{x}\right)$ cannot be smaller than 0 in the domain $(0, 12, 1)$

\therefore There is no subinterval in $(0, 12, 1)$ that the graph g is concave down

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Continue problem 2 on page 7.

Work for problem 2(c)

$$\therefore g'(x) = \sin\left(x + \frac{1}{x}\right)$$

$$\therefore g(x) = \int \sin\left(x + \frac{1}{x}\right) dx$$

$$\text{let } u = x + \frac{1}{x}$$

$$\therefore du =$$

let line tangent is $y = ax + b$

$$\therefore a = g'(x) = \sin\left(x + \frac{1}{x}\right)$$

$$\therefore x = 0.3$$

$$\therefore a = g'(0.3) = -0.472$$

$$\therefore y = -0.472x + b$$

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Work for problem 2(d)

$$g'(0.3) = -0.472$$

$$g'(1) = 0.909$$

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AP[®] CALCULUS AB
2010 SCORING COMMENTARY (Form B)

Question 2

Sample: 2A

Score: 9

The student earned all 9 points.

Sample: 2B

Score: 6

The student earned 6 points: 2 points in part (a), 2 points in part (b), 1 point in part (c), and 1 point in part (d). In parts (a) and (b), the student's work is correct. In part (c) the student earned the slope point for $g'(0.3)$. In part (d) the student's work is correct.

Sample: 2C

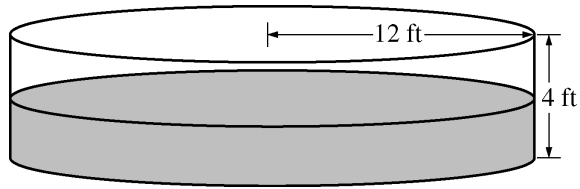
Score: 3

The student earned 3 points: 2 points in part (a), no points in part (b), 1 point in part (c), and no points in part (d). In part (a) the student's work is correct. In part (b) the student's concavity statement is incorrect. In part (c) the student earned the slope point for $g'(0.3)$. In part (d) the student does not include a statement about the tangent line.

AP[®] CALCULUS AB
2010 SCORING GUIDELINES (Form B)

Question 3

t	0	2	4	6	8	10	12
$P(t)$	0	46	53	57	60	62	63



The figure above shows an aboveground swimming pool in the shape of a cylinder with a radius of 12 feet and a height of 4 feet. The pool contains 1000 cubic feet of water at time $t = 0$. During the time interval $0 \leq t \leq 12$ hours, water is pumped into the pool at the rate $P(t)$ cubic feet per hour. The table above gives values of $P(t)$ for selected values of t . During the same time interval, water is leaking from the pool at the rate $R(t)$ cubic feet per hour, where $R(t) = 25e^{-0.05t}$. (Note: The volume V of a cylinder with radius r and height h is given by $V = \pi r^2 h$.)

- (a) Use a midpoint Riemann sum with three subintervals of equal length to approximate the total amount of water that was pumped into the pool during the time interval $0 \leq t \leq 12$ hours. Show the computations that lead to your answer.
- (b) Calculate the total amount of water that leaked out of the pool during the time interval $0 \leq t \leq 12$ hours.
- (c) Use the results from parts (a) and (b) to approximate the volume of water in the pool at time $t = 12$ hours. Round your answer to the nearest cubic foot.
- (d) Find the rate at which the volume of water in the pool is increasing at time $t = 8$ hours. How fast is the water level in the pool rising at $t = 8$ hours? Indicate units of measure in both answers.

(a) $\int_0^{12} P(t) dt \approx 46 \cdot 4 + 57 \cdot 4 + 62 \cdot 4 = 660 \text{ ft}^3$

2 : $\left\{ \begin{array}{l} 1 : \text{midpoint sum} \\ 1 : \text{answer} \end{array} \right.$

(b) $\int_0^{12} R(t) dt = 225.594 \text{ ft}^3$

2 : $\left\{ \begin{array}{l} 1 : \text{integral} \\ 1 : \text{answer} \end{array} \right.$

(c) $1000 + \int_0^{12} P(t) dt - \int_0^{12} R(t) dt = 1434.406$

1 : answer

At time $t = 12$ hours, the volume of water in the pool is approximately 1434 ft^3 .

(d) $V'(t) = P(t) - R(t)$
 $V'(8) = P(8) - R(8) = 60 - 25e^{-0.4} = 43.241$ or $43.242 \text{ ft}^3/\text{hr}$

$$V = \pi(12)^2 h$$

$$\frac{dV}{dt} = 144\pi \frac{dh}{dt}$$

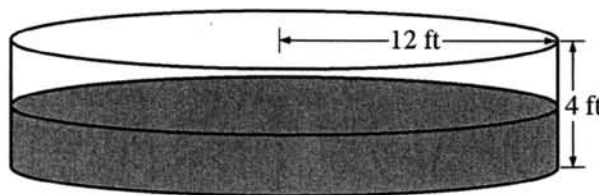
$$\left. \frac{dh}{dt} \right|_{t=8} = \frac{1}{144\pi} \cdot \left. \frac{dV}{dt} \right|_{t=8} = 0.095 \text{ or } 0.096 \text{ ft/hr}$$

4 : $\left\{ \begin{array}{l} 1 : V'(8) \\ 1 : \text{equation relating } \frac{dV}{dt} \text{ and } \frac{dh}{dt} \\ 1 : \left. \frac{dh}{dt} \right|_{t=8} \\ 1 : \text{units of } \text{ft}^3/\text{hr} \text{ and } \text{ft/hr} \end{array} \right.$

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3A,

t	0	2	4	6	8	10	12
$P(t)$	0	46	53	57	60	62	63



Work for problem 3(a)

$$\text{WATER ADDED INTO POOL} = \int_0^{12} P(t) dt \approx 4(46 + 57 + 62) = \boxed{660 \text{ ft}^3}$$

ABOUT 660 ft^3 OF WATER ARE ADDED TO THE POOL FROM $t = 0 \text{ h}$ TO $t = 12 \text{ h}$

Work for problem 3(b)

$$\text{WATER LEAVED} = \int_0^{12} R(t) dt = \int_0^{12} (25e^{-.05t}) dt = \boxed{225.594 \text{ ft}^3}$$

225.594 ft^3 OF WATER LEAVES FROM THE POOL FROM $t = 0 \text{ h}$ TO $t = 12 \text{ h}$

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Continue problem 3 on page 9.

Work for problem 3(c)

$$V = (\text{INITIAL WATER}) + (\text{WATER IN}) - (\text{WATER OUT}) = 1000 + \int_0^{12} P(t) dt - \int_0^{12} R(t) dt = 1660 - 225.594 =$$

$$\rightarrow = 1434.406 \text{ ft}^3$$

THE VOLUME OF WATER IN THE POOL
AT TIME $t = 12 \text{ h}$ IS ABOUT 1434 ft³

$$1434 \text{ ft}^3$$

Work for problem 3(d)

$$V = \pi r^2 h$$

$$\frac{dV}{dt} = \pi r^2 \frac{dh}{dt}$$

$$43.242 = 452389 \frac{dh}{dt}$$

$$\frac{dh}{dt} = .096 \text{ ft/hour}$$

$$\frac{dV}{dt} = (\text{RATE WATER IN}) - (\text{RATE WATER OUT})$$

$$\frac{dV}{dt} = P(8) - R(8) = 43.242 \text{ ft}^3/\text{hour}$$

AT $t = 8 \text{ h}$, THE VOLUME IN THE TANK
IS INCREASING AT 43.242 ft³/hour

AT $t = 8 \text{ h}$, THE WATER LEVEL IS RISING
AT .096 ft/hour

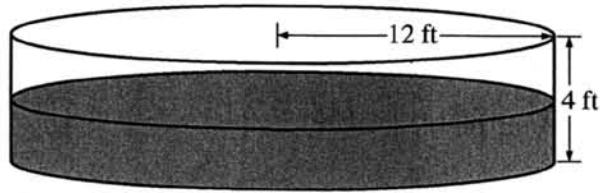
END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON
PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

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3B

t	0	2	4	6	8	10	12
$P(t)$	0	46	53	57	60	62	63



Work for problem 3(a)

Midpoints are $x=2, 6, 10$, $p(t)=46, 57, 62$.
 $Sum = 4(46+57+62) = 660$.

Work for problem 3(b)

$$V_{\text{water leaking out}} = \int_0^{12} R(t) \cdot dt = \int_0^{12} 25e^{-0.05t} \cdot dt$$

$$= 225.594$$

Continue problem 3 on page 9.

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Work for problem 3(c)

$$\begin{aligned}
 V &= V_{\text{pumped}} - V_{\text{leak}} \\
 &= 660 - 225.6 = 434.4 \text{ cubic feet} \\
 &\approx 434 \text{ cubic feet}
 \end{aligned}$$

Work for problem 3(d)

$$\begin{aligned}
 \frac{d}{dt} [p(t) - R(t)] &= \frac{d}{dt} (80 - 25e^{-0.05t}) \\
 &= -25 \cdot (-0.05) e^{-0.05t}
 \end{aligned}$$

$$\begin{aligned}
 \text{since } t=8 &\rightarrow +25 \times 0.05 \times 8 \times e^{-0.05 \times 8} \\
 &= 2.467
 \end{aligned}$$

Thus, the volume of water is increasing at the rate of 2.467 cubic feet/h at $t=8$

$$\begin{aligned}
 V &= \pi r^2 h \\
 \frac{dV}{dt} &= \pi r^2 \frac{dh}{dt} \\
 \frac{dh}{dt} &= \frac{2.467}{\pi \cdot 12^2} = 0.0055 \text{ ft/h}
 \end{aligned}$$

The water level is rising at the rate of 0.0055 ft/h at $t=8$.

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

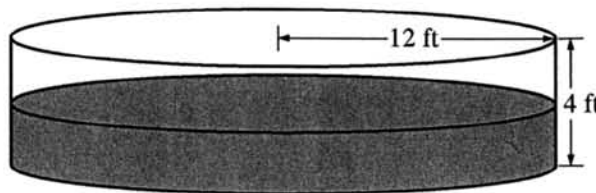
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t	0	2	4	6	8	10	12
$P(t)$	0	46	53	57	60	62	63



Work for problem 3(a)

The approximate total amount of water

$$= \frac{(0+53) \times 4}{2} + \frac{(53+60) \times 4}{2} + \frac{(60+63) \times 4}{2}$$

$$= 289 \times 2$$

$$= 578 \text{ cubic feet}$$

Work for problem 3(b)

The total amount of water leaking out

$$= \int_0^{12} 25e^{-0.05t} dt = 255.594 \text{ cubic feet}$$

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Continue problem 3 on page 9.

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3C₂

Work for problem 3(c)

The approximate volume of water in time $t=12$

$$= 1000 + 578 - 255.594$$

$$= 1322 \text{ cubic feet}$$

Work for problem 3(d)

The rate = $\frac{d}{dt} (60 - 25e^{-0.01t}) = 1.25 \cdot e^{-0.01t}$

$$= 0.8379 \text{ cubic feet per second}$$

The rate of rising = $\frac{d}{dt} (1.25 \cdot e^{-0.01t})$

$$= -0.0625 \cdot e^{-0.01t}$$

$$= -0.419 \text{ cubic feet per square second}$$

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON
PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

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AP[®] CALCULUS AB
2010 SCORING COMMENTARY (Form B)

Question 3

Sample: 3A

Score: 9

The student earned all 9 points.

Sample: 3B

Score: 6

The student earned 6 points: 2 points in part (a), 2 points in part (b), no points in part (c), and 2 points in part (d). In parts (a) and (b), the student's work is correct. In part (c) the student does not use the initial condition, and the point was not earned. In part (d) the student's presented value for $V'(8)$ is incorrect. The relationship between $\frac{dV}{dt}$ and $\frac{dh}{dt}$ is correct, and the value of $\frac{dh}{dt}$ is consistent with the student's $V'(8)$. The second and third points were earned. The units on $\frac{dh}{dt}$ are incorrect.

Sample: 3C

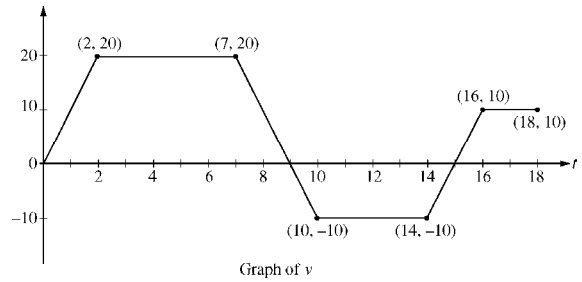
Score: 3

The student earned 3 points: no points in part (a), 2 points in part (b), 1 point in part (c), and no points in part (d). In part (a) the student does not use a midpoint Riemann sum. In part (b) the student's work is correct. In part (c) the student correctly combines the results from parts (a) and (b) along with the initial condition. In part (d) the student's work is incorrect.

AP[®] CALCULUS AB
2010 SCORING GUIDELINES (Form B)

Question 4

A squirrel starts at building A at time $t = 0$ and travels along a straight wire connected to building B . For $0 \leq t \leq 18$, the squirrel's velocity is modeled by the piecewise-linear function defined by the graph above.



- (a) At what times in the interval $0 < t < 18$, if any, does the squirrel change direction? Give a reason for your answer.
- (b) At what time in the interval $0 \leq t \leq 18$ is the squirrel farthest from building A ? How far from building A is the squirrel at this time?
- (c) Find the total distance the squirrel travels during the time interval $0 \leq t \leq 18$.
- (d) Write expressions for the squirrel's acceleration $a(t)$, velocity $v(t)$, and distance $x(t)$ from building A that are valid for the time interval $7 < t < 10$.

(a) The squirrel changes direction whenever its velocity changes sign. This occurs at $t = 9$ and $t = 15$.

2 : $\left\{ \begin{array}{l} 1 : t\text{-values} \\ 1 : \text{explanation} \end{array} \right.$

(b) Velocity is 0 at $t = 0$, $t = 9$, and $t = 15$.

2 : $\left\{ \begin{array}{l} 1 : \text{identifies candidates} \\ 1 : \text{answers} \end{array} \right.$

t	position at time t
0	0
9	$\frac{9+5}{2} \cdot 20 = 140$
15	$140 - \frac{6+4}{2} \cdot 10 = 90$
18	$90 + \frac{3+2}{2} \cdot 10 = 115$

The squirrel is farthest from building A at time $t = 9$; its greatest distance from the building is 140.

(c) The total distance traveled is $\int_0^{18} |v(t)| dt = 140 + 50 + 25 = 215$.

1 : answer

(d) For $7 < t < 10$, $a(t) = \frac{20 - (-10)}{7 - 10} = -10$

4 : $\left\{ \begin{array}{l} 1 : a(t) \\ 1 : v(t) \\ 2 : x(t) \end{array} \right.$

$$v(t) = 20 - 10(t - 7) = -10t + 90$$

$$x(7) = \frac{7+5}{2} \cdot 20 = 120$$

$$x(t) = x(7) + \int_7^t (-10u + 90) du$$

$$= 120 + (-5u^2 + 90u) \Big|_{u=7}^{u=t}$$

$$= -5t^2 + 90t - 265$$

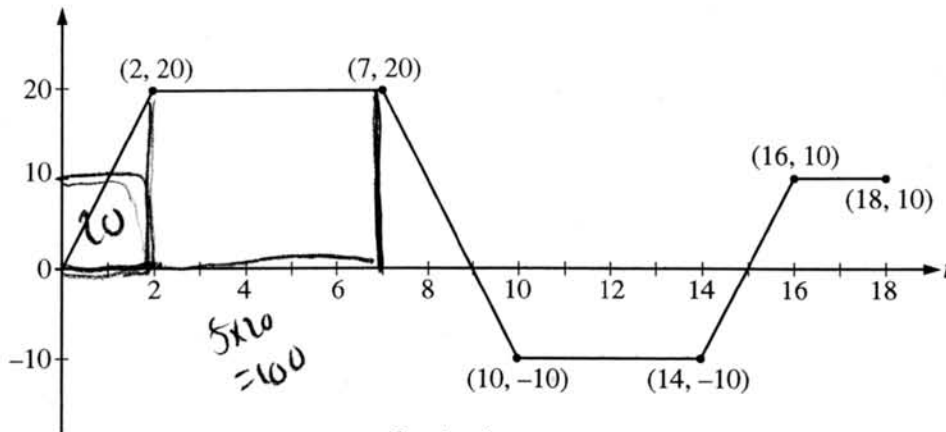
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CALCULUS BC
SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Graph of v

Work for problem 4(a)

The squirrel changes direction for $t=15$ and $t=9$ because velocity changes from negative to positive and vice versa on those points.

Work for problem 4(b) distance of squirrel from A, at $t: S(t)$

$$S(9) = \int_0^9 v(t) dt = 140$$

$$S(15) = \int_0^{15} v(t) dt = 140 - 50 = 90$$

$$S(18) = \int_0^{18} v(t) dt = 90 + 25 = 115$$

\therefore The squirrel is farthest from the building when $t=9$. The squirrel is 140 away from the building A.

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Continue problem 4 on page 11.

NO CALCULATOR ALLOWED

Work for problem 4(c)

$$\int_0^{18} |v(t)| = 140 + 50 + 25 = \underline{215}$$

Work for problem 4(d)

in (7, 10)

$$a(t) = v'(t) = \frac{-10 - 20}{10 - 7} = \frac{-30}{3} = \underline{-10}$$

$$v(9) = 0$$

$$\text{velocity: } y - 0 = -10(x - 9)$$

$$y = -10x + 90$$

$$\therefore v(t) = \underline{-10x + 90}$$

$$x(t) = x(7) + \int_7^t v(t) dt$$

$$= 120 + [-5x^2 + 90x]_7^t$$

$$= 120 + 5t^2 + 90t - (385)$$

$$= \underline{-5t^2 + 90t - 265}$$

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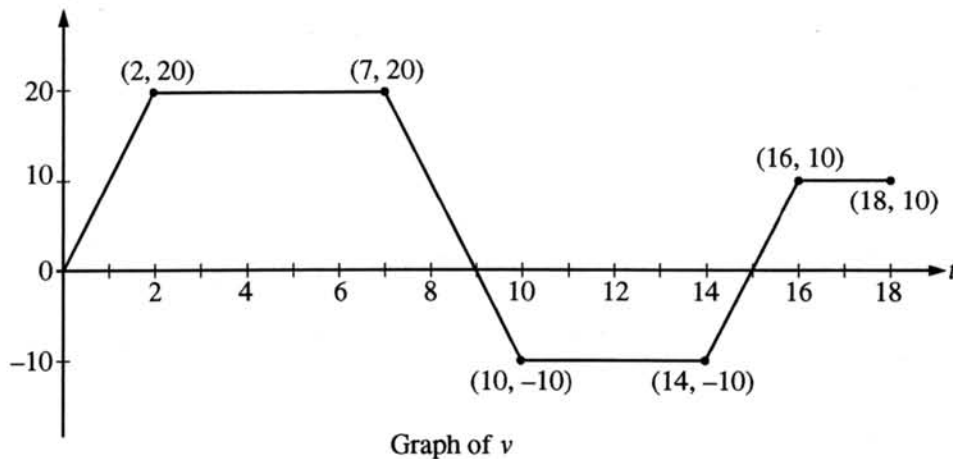
CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Work for problem 4(a)

The squirrel changes direction at $t = 9$ and $t = 15$. His velocity changes from positive to negative.

Work for problem 4(b)

At $t = 9$ the squirrel is farthest from the building A. At $t = 9$, the squirrel is 140 units away from building A.

$$\frac{1}{2} \cdot 20 \cdot (9 + 5) = 140$$

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Continue problem 4 on page 11.

NO CALCULATOR ALLOWED

Work for problem 4(c)

$$\frac{1}{2} \cdot 20 \cdot (14) + \frac{1}{2} \cdot 10 \cdot (2+3) + \frac{1}{2} \cdot 10 \cdot (6+4)$$

$$140 + 25 + 50 = 215$$

Total distance traveled = 215 units.

Work for problem 4(d)

$$\frac{-10 - 20}{10 - 7} =$$

$$\frac{-30}{3} = -10$$

$$v(t) = -10x + 90$$

$$x(t) = -5x^2 + 90x + 120$$

$$c = \frac{1}{2} \cdot 20 \cdot (5+7) \int -10x + 90 dx$$

$$c = 120 \quad -5x^2 + 90x + C$$

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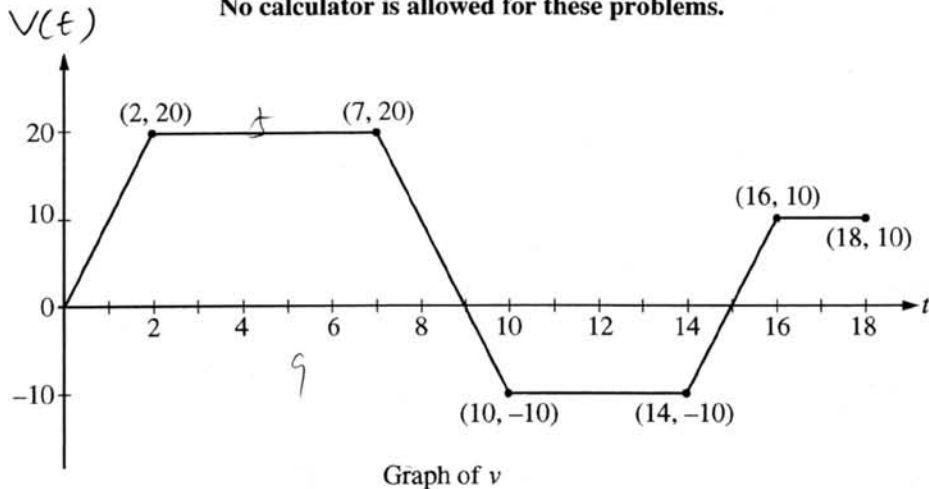
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NO CALCULATOR ALLOWED

**CALCULUS BC
SECTION II, Part B**

**Time—45 minutes
Number of problems—3**

No calculator is allowed for these problems.



Work for problem 4(a)

at $9 < t < 15$, the squirrel changes its direction since its velocity changes from positive to negative.

Work for problem 4(b)

1) at $t = 9$. because ^{that's when} the area between the graph of $v(t)$ and the x -axis is the largest.

$$2) S = \frac{(5+9) \times 20}{2} = 140$$

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Continue problem 4 on page 11.

NO CALCULATOR ALLOWED

Work for problem 4(c)

Total Distance :

$$= \left| \int_0^9 v(t) dt \right| - \left| \int_9^{15} v(t) dt \right| + \left| \int_{15}^{18} v(t) dt \right|$$

$$= 140 - 50 + 25$$

$$= 115$$

Work for problem 4(d)

$v(t)$ @ $7 < t < 10$ is a straight line.

passing $(7, 20), (10, -10)$

$$\therefore v(t) = \underline{-10t + 90.}$$

According to motion Theorems,

$$a(t) = v'(t) = \underline{-10}$$

$$x(t) = \int v(t) dt = \underline{-5t^2 + 90t}$$

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AP[®] CALCULUS AB
2010 SCORING COMMENTARY (Form B)

Question 4

Sample: 4A

Score: 9

The student earned all 9 points.

Sample: 4B

Score: 6

The student earned 6 points: 1 point in part (a), 1 point in part (b), 1 point in part (c), and 3 points in part (d). In part (a) the student identifies the two points at which the graph of v crosses the t -axis but does not correctly explain why the squirrel changes direction at those two points. The given explanation applies to only one of the two points. In part (b) the student does not identify all candidates but does evaluate the distance at $t = 9$. The second point was earned. In part (c) the student's work is correct. In part (d) the student has correct expressions for $a(t)$ and $v(t)$, but the expression for $x(t)$ does not incorporate the initial condition. One of the points for $x(t)$ was earned.

Sample: 4C

Score: 3

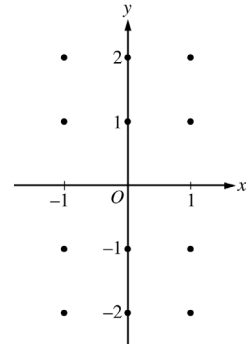
The student earned 3 points: no points in part (a), 1 point in part (b), no points in part (c), and 2 points in part (d). In part (a) the student presents an interval instead of points. In part (b) the student does not identify all candidates but does evaluate the distance at $t = 9$. The second point was earned. In part (c) the student finds displacement rather than total distance traveled. In part (d) the student has correct expressions for $a(t)$ and $v(t)$ but not for $x(t)$.

AP[®] CALCULUS AB
2010 SCORING GUIDELINES (Form B)

Question 5

Consider the differential equation $\frac{dy}{dx} = \frac{x+1}{y}$.

- (a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated, and for $-1 < x < 1$, sketch the solution curve that passes through the point $(0, -1)$.



(Note: Use the axes provided in the exam booklet.)

- (b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy -plane for which $y \neq 0$. Describe all points in the xy -plane, $y \neq 0$, for which $\frac{dy}{dx} = -1$.
- (c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = -2$.

(a)

3 : $\left\{ \begin{array}{l} 1 : \text{zero slopes} \\ 1 : \text{nonzero slopes} \\ 1 : \text{solution curve through } (0, -1) \end{array} \right.$

(b) $-1 = \frac{x+1}{y} \Rightarrow y = -x - 1$

$\frac{dy}{dx} = -1$ for all (x, y) with $y = -x - 1$ and $y \neq 0$

1 : description

(c) $\int y \, dy = \int (x+1) \, dx$

$\frac{y^2}{2} = \frac{x^2}{2} + x + C$

$\frac{(-2)^2}{2} = \frac{0^2}{2} + 0 + C \Rightarrow C = 2$

$y^2 = x^2 + 2x + 4$

Since the solution goes through $(0, -2)$, y must be negative. Therefore $y = -\sqrt{x^2 + 2x + 4}$.

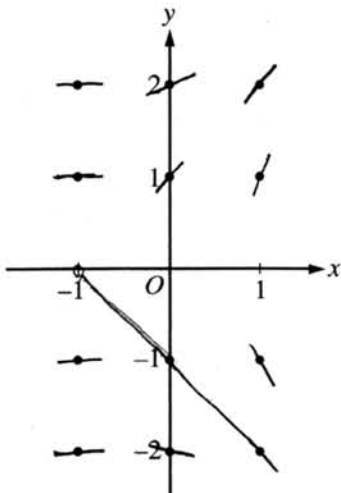
5 : $\left\{ \begin{array}{l} 1 : \text{separates variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{array} \right.$

Note: max 2/5 [1-1-0-0-0] if no constant of integration

Note: 0/5 if no separation of variables

NO CALCULATOR ALLOWED

Work for problem 5(a)



Work for problem 5(b)

$$\frac{dy}{dx} = \frac{x+1}{y} = -1 \quad \Rightarrow \quad x+1 = -y \quad \Rightarrow \quad x+y = -1$$

except point $(-1, 0)$, points $(x, -1-x)$

are all solution to $\frac{dy}{dx} = -1$.

They are on the line $y = -1-x$.

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Continue problem 5 on page 13.

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5A₂

NO CALCULATOR ALLOWED

Work for problem 5(c)

$$\frac{dy}{dx} = \frac{x+1}{y}$$

$$dy \cdot y = dx \cdot (x+1)$$

$$\int y \cdot dy = \int (x+1) dx$$

$$\frac{1}{2}y^2 = \frac{1}{2}x^2 + x + C \quad \text{passes through } f(0) = -2$$

$$\frac{1}{2} \times 4 = C \Rightarrow C = 2$$

$$y = -\sqrt{x^2 + 2x + 4}$$

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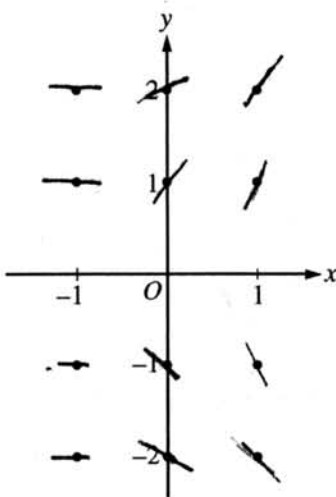
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5B

NO CALCULATOR ALLOWED

Work for problem 5(a)



Work for problem 5(b)

$$\frac{dy}{dx} = -1 \text{ when}$$

$$0 \leq x \leq 1 \text{ and } -2 \leq y \leq -1$$

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Continue problem 5 on page 13.

NO CALCULATOR ALLOWED

Work for problem 5(c)

$$\frac{dy}{dx} = \frac{x+1}{y} \quad f(0) = 2$$

$$y dy = (x+1) dx$$

$$\frac{y^2}{2} = \frac{x^2}{2} + x + C$$

$$\frac{(-2)^2}{2} = \frac{0^2}{2} + 0 + C$$

$$\frac{4}{2} = C$$

$$2 = C$$

$$\frac{y^2}{2} = \frac{x^2}{2} + x + 2$$

$$y^2 = 2\left(\frac{x^2}{2} + x + 2\right)$$

$$y^2 = x^2 + 2x + 4$$

$$y = \sqrt{x^2 + 2x + 4}$$

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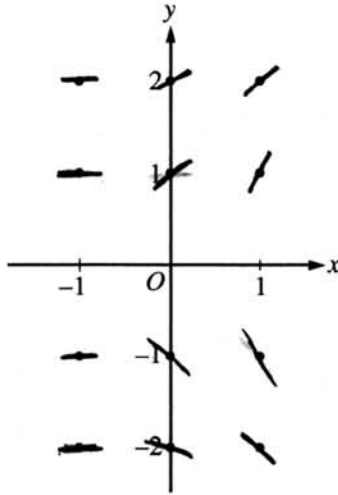
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5c

NO CALCULATOR ALLOWED

Work for problem 5(a)



Work for problem 5(b)

$$\underline{\underline{y = -x}}$$

$\frac{dy}{dx} = -1$ means that the slope is equal to -1 .

For all x , if $f(x) = -x$ $\frac{dy}{dx} = -1$

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border.

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Continue problem 5 on page 13.

NO CALCULATOR ALLOWED

Work for problem 5(c)

$$\frac{dy}{dx} = \frac{x+1}{y}$$

$$y \cdot dy = dx(x+1)$$

$$\int y \cdot dy = \int dx(x+1)$$

$$y = x + 1$$

$$f(0) = -2$$

$$dx = -2dy$$

$$\int dx = \int -2dy$$

$$x = -2y$$

$$y = f\left(-\frac{x}{2}\right)$$

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AP[®] CALCULUS AB
2010 SCORING COMMENTARY (Form B)

Question 5

Sample: 5A

Score: 9

The student earned all 9 points.

Sample: 5B

Score: 6

The student earned 6 points: 2 points in part (a), no points in part (b), and 4 points in part (c). In part (a) the student's slope field is correct, but no solution curve is given. In part (b) the student's description is incorrect. In part (c) the student earned the first 4 points. Although the student uses the initial condition, the incorrect branch is chosen for the solution.

Sample: 5C

Score: 3

The student earned 3 points: 2 points in part (a), no points in part (b), and 1 point in part (c). In part (a) the student's slope field is correct, but no solution curve is given. In part (b) the student's description is incorrect. In part (c) the student earned the point for separation of variables. The antiderivatives are not correct, so the student was not eligible for additional points.

AP[®] CALCULUS AB
2010 SCORING GUIDELINES (Form B)

Question 6

Two particles move along the x -axis. For $0 \leq t \leq 6$, the position of particle P at time t is given by

$$p(t) = 2 \cos\left(\frac{\pi}{4}t\right), \text{ while the position of particle } R \text{ at time } t \text{ is given by } r(t) = t^3 - 6t^2 + 9t + 3.$$

- (a) For $0 \leq t \leq 6$, find all times t during which particle R is moving to the right.
 (b) For $0 \leq t \leq 6$, find all times t during which the two particles travel in opposite directions.
 (c) Find the acceleration of particle P at time $t = 3$. Is particle P speeding up, slowing down, or doing neither at time $t = 3$? Explain your reasoning.
 (d) Write, but do not evaluate, an expression for the average distance between the two particles on the interval $1 \leq t \leq 3$.

(a) $r'(t) = 3t^2 - 12t + 9 = 3(t-1)(t-3)$
 $r'(t) = 0$ when $t = 1$ and $t = 3$
 $r'(t) > 0$ for $0 < t < 1$ and $3 < t < 6$
 $r'(t) < 0$ for $1 < t < 3$

Therefore R is moving to the right for $0 < t < 1$ and $3 < t < 6$.

(b) $p'(t) = -2 \cdot \frac{\pi}{4} \sin\left(\frac{\pi}{4}t\right)$
 $p'(t) = 0$ when $t = 0$ and $t = 4$
 $p'(t) < 0$ for $0 < t < 4$
 $p'(t) > 0$ for $4 < t < 6$

Therefore the particles travel in opposite directions for $0 < t < 1$ and $3 < t < 4$.

(c) $p''(t) = -2 \cdot \frac{\pi}{4} \cdot \frac{\pi}{4} \cos\left(\frac{\pi}{4}t\right)$
 $p''(3) = -2\left(\frac{\pi}{4}\right)^2 \cos\left(\frac{3\pi}{4}\right) = \frac{\pi^2}{8} \cdot \frac{\sqrt{2}}{2} > 0$
 $p'(3) < 0$

Therefore particle P is slowing down at time $t = 3$.

(d) $\frac{1}{2} \int_1^3 |p(t) - r(t)| dt$

$$2 : \begin{cases} 1 : r'(t) \\ 1 : \text{answer} \end{cases}$$

$$3 : \begin{cases} 1 : p'(t) \\ 1 : \text{sign analysis for } p'(t) \\ 1 : \text{answer} \end{cases}$$

$$2 : \begin{cases} 1 : p''(3) \\ 1 : \text{answer with reason} \end{cases}$$

$$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and constant} \end{cases}$$

NO CALCULATOR ALLOWED

Work for problem 6(a)

Particle R is moving to the right when $r'(t) > 0$

$$r'(t) = 3t^2 - 12t + 9 = 3 \cdot (t^2 - 4t + 3) \quad t=1, t=3 \quad r'(t) = 0$$

$$t = 1, t = 3. \quad r'(t) > 0 \text{ for } t \in (0, 1) \cup (3, 6)$$

Work for problem 6(b)

$$p(t) = 2 \cos\left(\frac{\pi}{4}t\right) \quad p'(t) = -2 \sin\left(\frac{\pi}{4}t\right) \cdot \frac{\pi}{4} =$$

$$= -\frac{\pi}{2} \sin\left(\frac{\pi}{4}t\right) \quad p'(t) = 0 \quad t = 4 \quad p'(t) > 0 \text{ for } t \in (4, 6)$$

and $p'(t) < 0$ for $t \in (0, 4)$ In order for the particles to move in different directions:

~~$p'(t) > 0 \quad t \in (4, 6) \quad p'(t) > 0 \quad t \in (0, 1) \cup (3, 6)$~~

they move both p'

$$\begin{cases} p'(t) > 0 \\ r'(t) < 0 \end{cases} \Leftrightarrow \begin{cases} t \in (0, 1) \cup (3, 6) \\ t \in (4, 6) \end{cases} \Leftrightarrow t \in (0, 1) \cup (3, 4)$$

$$\begin{cases} r'(t) > 0 \\ p'(t) < 0 \end{cases} \Leftrightarrow \begin{cases} t \in (0, 1) \cup (3, 6) \\ t \in (0, 4) \end{cases} \Leftrightarrow t \in (0, 1) \cup (3, 4)$$

Thus they move in different directions for $t \in (0, 1) \cup (3, 4)$

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NO CALCULATOR ALLOWED

Work for problem 6(c)

$$p(t) = 2 \cos\left(\frac{\pi}{4}t\right) \quad p'(t) = -2 \sin\left(\frac{\pi}{4}t\right) \cdot \frac{\pi}{4} = -\frac{\pi}{2} \sin\left(\frac{\pi}{4}t\right)$$

$$p''(t) = (p'(t))' = -\frac{\pi}{8} \cos\left(\frac{\pi}{4}t\right) \quad p''(3) = -\frac{\pi^2}{8} \cos\left(\frac{3}{4}\pi\right) = \frac{\pi^2}{8\sqrt{2}}$$

as $p''(t) > 0$ the particle P is speeding up at $t=3$.

$$p'(3) = -\frac{\pi}{2} \sin\left(\frac{3}{4}\pi\right) = -\frac{\pi}{2\sqrt{2}}$$

as $p'(3) < 0$ and $p''(3) > 0$ the particle is slowing down at $t=3$.

Work for problem 6(d)

$$\frac{1}{3-1} \int_1^3 |p(t) - r(t)| dt = \frac{1}{2} \int_1^3 |p(t) - r(t)| dt$$

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NO CALCULATOR ALLOWED

Work for problem 6(a)

$$r(t) > 0$$

$$r(t) = 3t^2 - 12t + 9 > 0$$

$$3(t^2 - 4t + 3) > 0$$

$$3(t-3)(t-1) > 0$$

$$t > 1, t > 3$$

$$3 < t \leq 6$$

Work for problem 6(b)

$$p'(t) = -\frac{\pi}{2} \sin\left(\frac{\pi}{4}t\right) \quad p'(t) < 0 \quad [0, 4]$$

$$p'(t) > 0 \quad [4, 6]$$

whereas

$$r'(t) < 0 \quad [0, 3]$$

$$r'(t) > 0 \quad (3, 6]$$

Particles R and P are moving in opposite directions from time $3 < t \leq 4$

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Continue problem 6 on page 15.

Work for problem 6(c)

$$p''(t) = a(t)$$

$$p'(t) = -2 \sin\left(\frac{\pi}{4}t\right) \cdot \frac{\pi}{4} \cdot 2$$

$$p'(t) = -\frac{\pi}{2} \sin\left(\frac{\pi}{4}t\right)$$

$$p''(t) = -\frac{\pi}{2} \cos\left(\frac{\pi}{4}t\right) \cdot \frac{\pi}{4}$$

$$= -\frac{\pi^2}{8} \cos\left(\frac{\pi}{4}t\right)$$

$$p''(3) = -\frac{\pi^2}{8} \cos\left(\frac{3}{4}\pi\right)$$

$$\cos\left(\frac{3}{4}\pi\right) < 0 \therefore p''(3) > 0 \therefore \text{speeding up.}$$

Work for problem 6(d)

$$\frac{1}{b-a} \int_a^b r(t) - p(t) dt$$

$$\frac{1}{2} \int_1^3 t^3 - 6t^2 + 9t + 3 - 2 \cos\left(\frac{\pi}{4}t\right) dt$$

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Work for problem 6(a)

$$r(t) = t^3 - 6t^2 + 9t + 3$$

$$r'(t) = 3t^2 - 12t + 9$$

$$\begin{array}{r|l} t & -3 \\ \times & 3 \\ \hline & -3t \\ & +9 \\ \hline t^2 + 3 & -4t \end{array}$$

$$r'(t) = 0$$

$$3t^2 - 12t + 9 = 0$$

$$t^2 - 4t + 3 = 0$$

$$(t-3)(t-1) = 0$$

$$t = 3 \text{ or } t = 1$$

Since R is moving to the right,
 $r(t) > 0$

$$t = 3 \text{ and } t = 1$$

$$r(1) = 1 - 6 + 9 + 3$$

$$= 7$$

$$r(3) = 27 - 6(9) + 9(3) + 3$$

$$= 27 - 54 + 27 + 3$$

$$= 3$$

Work for problem 6(b)

$$p(t) = 2 \cos\left(\frac{\pi}{4}t\right)$$

$$p'(t) = -2 \sin\left(\frac{\pi}{4}t\right) \left(\frac{\pi}{4}\right)$$

$$= -\frac{\pi}{2} \sin\left(\frac{\pi}{4}t\right)$$

When 2 particles travel in opposite directions,

$$2 \cos\left(\frac{\pi}{4}t\right) = t^3 - 6t^2 + 9t + 3$$

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Continue problem 6 on page 15.

Work for problem 6(c)

$$P(t) = 2 \cos\left(\frac{\pi}{4}t\right)$$

$$\begin{aligned} P'(t) &= -2 \sin\left(\frac{\pi}{4}t\right) \left(\frac{\pi}{4}\right) \\ &= -\frac{\pi}{2} \sin\left(\frac{\pi}{4}t\right) \end{aligned}$$

$$\begin{aligned} A = P''(t) &= -\frac{\pi}{2} \cos\left(\frac{\pi}{4}t\right) \left(\frac{\pi}{4}\right) \\ &= -\frac{\pi}{2} \left(\frac{\pi}{4}\right) \cos\left(\frac{\pi}{4}t\right) \end{aligned}$$

When $t = 3$

$$\begin{aligned} A &= -\frac{\pi}{2} \left(\frac{\pi}{4}\right) \cos\left(\frac{3\pi}{4}\right) \\ &= -\frac{\pi^2}{8} \cos\left(\frac{3\pi}{4}\right) \end{aligned}$$

Particle P is slowing down because the acceleration of the particle is decreasing (< 0).

Work for problem 6(d)

$$\text{Av. distance} = \frac{1}{3-1} \left(\left[2 \cos \frac{\pi}{4}t \right]_1^3 + \left[t^3 - 6t^2 + 9t + 3 \right]_1^3 \right)$$

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AP[®] CALCULUS AB
2010 SCORING COMMENTARY (Form B)

Question 6

Sample: 6A

Score: 9

The student earned all 9 points.

Sample: 6B

Score: 6

The student earned 6 points: 1 point in part (a), 3 points in part (b), 1 point in part (c), and 1 point in part (d). In part (a) the student earned the point for $r'(t)$. Only one of the intervals is identified, so the second point was not earned. In part (b) the student's work is correct. The student's answer for when the two particles travel in opposite directions is consistent with the incorrect work in part (a); thus the point was earned. In part (c) the student earned the point for $p''(3)$, but the conclusion is not correct. In part (d) the student has the correct limits of integration and the correct constant factor but an incorrect integrand.

Sample: 6C

Score: 3

The student earned 3 points: 1 point in part (a), 1 point in part (b), 1 point in part (c), and no points in part (d). In part (a) the student earned the point for $r'(t)$. In part (b) the student earned the point for $p'(t)$. In part (c) the student earned the point for $p''(3)$. In part (d) the student does not provide an integral.